Short Papers

Analysis of Planar Structures by an Integral Approach Using Entire Domain Trial Functions

M. Nadarassin, H. Aubert, and H. Baudrand

***Abstract*—In this paper, a method based on an integral formulation with an excitation term is presented. The particularities of this approach lie in the utilization of entire domain trial functions and in the characterization of the coupling effect due to the excitation mechanism. The trial functions are taken as the TE-TM modes of a waveguide whose cross section corresponds to the shape of the discontinuity. The trial functions are computed and stored in the memory, then the study of complex planar structures becomes easy. A complete study is proposed, including the analysis of the coupling mechanism due to the source interaction and the characterization of higher order modes influence. This work is followed by two applications: a multiaxial discontinuity (bend discontinuity) in microstrip and in CPW. The computed results have been compared with data furnished by the literature. A good accuracy has been obtained.**

I. Introduction

To overcome the lack of accuracy of the traditional models based on a quasistatic analysis [1] or an equivalent waveguide consideration [2], different methods have been developed. However, as it was pointed out in [3], the CAD softwares which are commercially available do not take sufficiently the shielding effects of the circuit into account.

Methods based on an integral formulation seem to be accurate and rigorous tools for the treatment of this type of problem. Among these methods, we can distinguish methods lying in an iterative process of resolution, that is the resolution of an eigenvalue problem, and others which by the introduction of an excitation source reduce the equations into an inhomogeneous system via the application of the method of moments. With that purpose, excitation terms were largely introduced in the integral methods [3]–[5]. Moreover, the integral methods become efficient if trial functions have been correctly chosen: so an important attention must be paid to this choice in order to reduce the computation time with small system matrix. It's with this aim that we have developed in this paper, an integral method with entire domain trial functions has been developed. L-shaped trial functions expressed in the entire domain and obtained by an initial computation, allow the characterization of bend-discontinuities in microstrip lines and coplanar waveguides. This approach is quite flexible and allows a rigorous treatment of different structures (Tee, Gap, Bend, and Step). The implementation of our method is discussed and the numerical results have been compared with published data.

II. Theoretical Developments

A shielded structure is considered in the theoretical formulation. The circuits are assumed to be lossless with infinitely thin metallization and isotropic substrate. The circuits are placed in an electric wall cavity in the case of microstrip line and in a magnetic wall cavity in the case of CPW. The choice of the nature of the wall is made so that the compatibility with the excitation sources placed at the port is ensured.

To explain clearly the development of the theoretical formulation, we use to a large extent the formalism presented in [6] and [7]. For the same purpose we treat a simple case: microstrip line. Let J and E be the electric density of current and the electric field on the discontinuity plane S, respectively. J is defined according to the magnetic field H as $J = H \times n$, where n is the normal vector to the discontinuity plane. The integral equation associated with the problem is

$$\hat{Z}\, J = e_0 \tag{1}$$

where $\hat{Z}$ designates the impedance operator which characterizes the cavity and e_0 is the excitation term relative to a magnetic current M_0. The unknown quantity J is expanded on a basis (g_i) of trial functions which will be explained in more details in Section IV

$$J = \sum_i a_i g_i. \tag{2}$$

Next, the application of Galerkin's procedure transforms (1) into the following linear system of equations

$$\sum_j \langle g_i, \hat{Z} g_j\rangle\, a_j = \langle g_i, e_0\rangle, \quad \forall i \tag{3}$$

where the inner product is defined by

$$\langle f_m, f_n\rangle = \iint_S f_m \cdot f_n^*\, ds = \delta_{mn} \tag{4}$$

and δ_{mn} designates the delta Kronecker function. The resolution of the matrix system (3) permits the determination of the components a_j. Then, by using a variational formula [8], the input impedance Z_{in} is computed. This impedance value does not correspond to the actual value of the impedance [5], [9]. The computed impedance is the one viewed by the source. Therefore, a de-embedding technique must be developed to obtain the impedance viewed by the fundamental mode.

III. Coupling Effects Analysis

Many methods have been developed with the aim of extracting the scattering parameters. Among the techniques presented in the literature, we can distinguish techniques based on the analysis of computed current [4] and others lying in the computation of the input impedance [5] and [9]. In our work, different methods have been implemented, and finally, a modified approach of [9] has been used because of its flexibility and its accuracy. Our improvement concerns the nature of the excitation source e_0 and allows to avoid numerical divergence of results arising in [9]. The de-embedding process can be reduced to the determination of three parameters: A, B, and C of a homographic relation

$$Z_{in} = A + \frac{B}{1 + C.z} \tag{5}$$

where Z_{in} is the computed impedance and z denotes the impedance associated with the fundamental mode. It is clear that A, B, and C are determined by considering known termination conditions (perfect short-circuit or perfect open-end). Few computed results are sufficient to estimate their values. A least-squares method can also be used to minimize the error. Consequently, the modeling of other circuits

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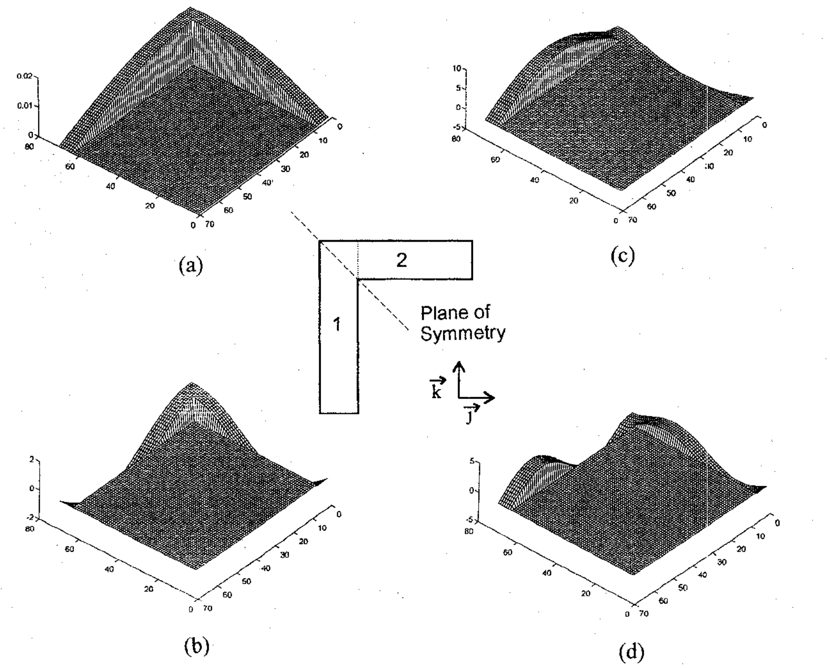

Fig. 1. Shape of the first four generating functions, (a) and (b) an electric wall inserted in plane of symmetry, (c) and (d) a magnetic wall inserted in plane of symmetry.

is made possible. The homographic relation can be represented by an equivalent network composed of a shunt impedance, a series impedance, and a transformer. In the approach presented in [5], a shunt impedance is placed at the port junction in order to take the disturbance effects of the excitation into account: homographic relation (5) characterizes this disturbance more accurately, especially at higher frequencies.

IV. Entire Domain Trial Functions

As a general rule, the TE-TM modes of a waveguide whose cross section corresponds to the shape of the planar circuit are taken as the trial functions basis. This basis is obtained by the resolution of the Helmholtz equation in this waveguide. In the case of simple structures, the latter is expanded in an analytical basis as a short circuited or open-ended line where the TE-TM modes of a rectangular guide are taken. In the case of L-shaped waveguide (see Fig. 1), methods have been developed for the computation of these modes [10] and no more refinements are needed within the scope of this paper. Each trial function g_i is expanded as below

$$g_i = \begin{cases} \sum_{n=1}^{N} g_{y_1}^n \vec{\jmath} + \sum_{m=1}^{M} g_{z_1}^m \vec{k} & \text{Region 1} \\ \sum_{n=1}^{N} g_{y_2}^n \vec{\jmath} + \sum_{m=1}^{M} g_{z_2}^m \vec{k} & \text{Region 2.} \end{cases} \tag{6}$$

The set of all these trial functions is used to expand the unknown current density. Note that a small number of functions gives very accurate numerical results: thus, this approach is not very time-consuming. The rooftop expansion functions approach is largely developed in the literature for complex discontinuities, as long as the structures can be divided into rectangles or triangles [4]. But this approach generates large matrix systems while our approach using entire domain trial functions needs very small matrices (see next section).

In the case of L-shaped trial functions, a wide class of discontinuities as tee, bend, and gap can be easily characterized. The modeling of such discontinuities is achieved by taking into account the different symmetries of these structures, so that the domain of study is easily reduced to L-shaped domains in each case and then, the current is expanded in L-shaped domains. For the computation of the trial functions, it is important to satisfy the boundary conditions and take advantage of the symmetries in order to reduce the computation effort. For instance, for a step or a tee discontinuity a magnetic wall is inserted in the plane of symmetry.

Since we have defined the trial functions, the scattering parameters are obtained by calculating two impedances: Z_o (odd-symmetry) and Z_e (even symmetry). These impedances are computed by placing respectively an electric wall or a magnetic wall in the plane of symmetry of the structure (Fig. 1). In other terms, we have to solve twice the system of linear equations (3), in which the trial functions have been selected with a particular symmetry. For instance, in the case of a bend discontinuity in microstrip line, we have to choose the corresponding trial functions: electric wall [Fig. 1(a) and 1(b)], magnetic wall [Fig. 1(c) and 1(d)]. With these two impedances corrected by using the relation (5), the impedance matrix is determined: $Z_{11} = (Z_e + Z_o)/2$ and $Z_{12} = (Z_e - Z_o)/2$. The S-parameters are immediately deduced.

For CPW structures, as a general rule, the electric field is taken as unknown and expanded on a basis of trial functions. In that case, the trial functions are obtained by computing the cutoff wavenumbers of a waveguide composed of electric walls. The calculation is analog to that of the microstrip structures, only the excitation term has been replaced by an impressed current j_0 defined on the slot.

V. Numerical Results

First, the method is implemented for a bend-discontinuity in microstrip line. A small number of trial functions (16)–(32) is sufficient to obtain the convergence of the results. The dimensions of the structure are the same as those used by [12]: w = 0.635 mm, h = 0.635 mm, a = 3.18 mm, b = c = 12.7 mm ε_r = 9.8. The scattering parameters are calculated for frequencies between 8–20 GHz. The results are compared with the analysis described in [12]

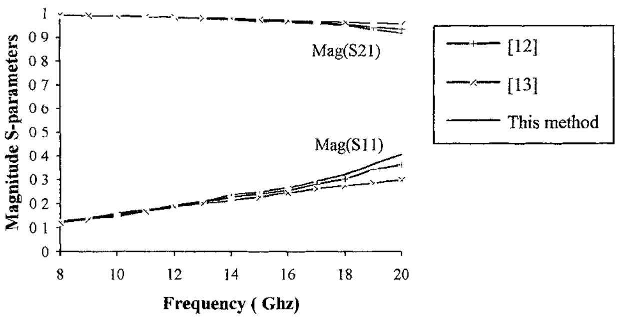

Fig. 2. Microstrip Bend-discontinuity. Magnitude of S-parameters.

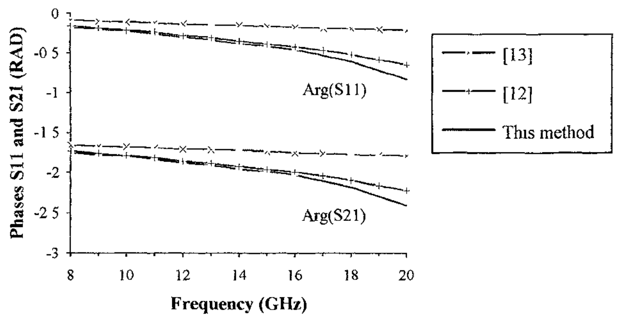

Fig. 3. Microstrip Bend-discontinuity. Phase of S-parameters.

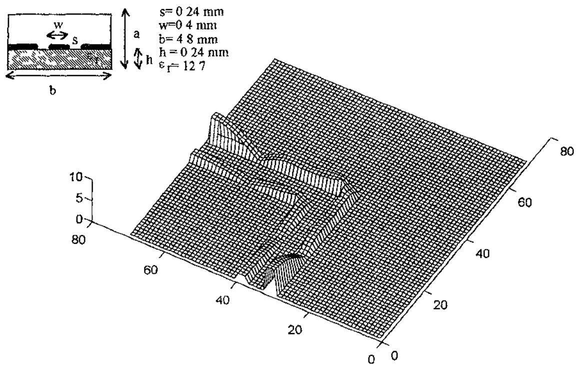

Fig. 4. CPW bend-discontinuity, magnitude of the transverse component of the electric field with a magnetic wall in the plane of symmetry.

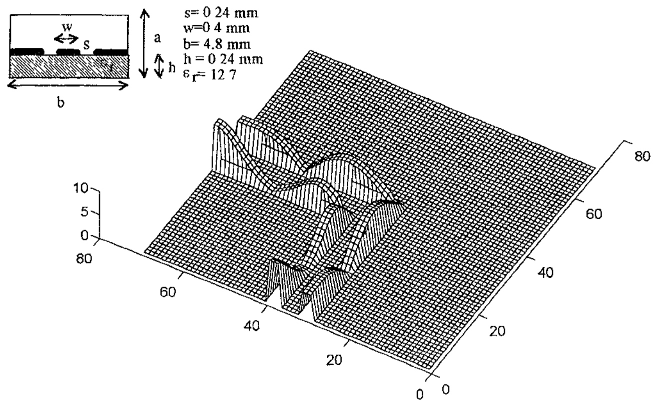

Fig. 5. CPW bend-discontinuity, magnitude of the transverse component of the electric field with an electric wall in the plane of symmetry.

and [13] (see Fig. 2). We can observe that in [13] the analysis does not take into account the shielding effects and lies in an open structure analysis, that is why the magnitude of S_{11} is always smaller than the results obtained in this analysis and in [12]. The difference between the curves increases at higher frequencies because of the box mode which is a propagating mode. The variations in phase of S_{12} and S_{11} are shown in Fig. 3. Although, the phase of S_{21} is smaller than the results published in [12], the difference is not very significant. Moreover, the tendency of these two curves confirms the importance of the shielding effects at higher frequencies.

The application of this method to a bend in coplanar waveguide is presented in Figs. 4 and 5. The figures show the shape of the magnitude of the computed current in the case where an electric wall is inserted in the plane of symmetry and in the case of a magnetic plane inserted in the plane of symmetry. With these two results, the impedance matrix is deduced, then the scattering parameters are also computed. One can observe the correctness of the results, at low frequencies, few trial functions are sufficient to obtain good results. At high frequencies, another difficulty appears with the second mode which propagates along the line. In that case a fourth port network is introduced to take into account the distribution in energy on the second mode. From the above numerical results, we notice that a limited number of entire domain trial functions is required to achieve accurate modeling and consequently the matrix dimensions are small in all cases. Moreover, the domain of definition of entire domain trial function is smaller than the one corresponding to the rooftops: thus, the number of terms in the modal expansion of each trial function is relatively small compared with the case of rooftops. Several improvements of our method can be introduced in this approach as in [4] in order to reduce the computation time: for instance, once the inner products involved in the calculation are stored in the memory, only the modal admittances have to be computed at each frequency. Although our numerical results concern a class of particular discontinuities (L-shaped discontinuities), the present approach can be easily developed for discontinuity of complex shape as long as the TE and TM modes of a waveguide, whose cross section corresponds to the discontinuity shape can be determined. A method of segmentation [11] allows this determination without increasing the matrix dimensions of the system (3).

VI. Conclusion

In this paper, a method based on an integral formulation is presented. As an application, two discontinuities are modeled by means of this method with entire domain trial functions, which are in fact the TE-TM modes of a waveguide whose cross section corresponds to the shape of the discontinuity. The accuracy of this analysis is proved, only few functions are sufficient to yield good results. Now, the computer code developed for this study is extended to others discontinuities like tee, gap, and step in microstrip line and CPW.

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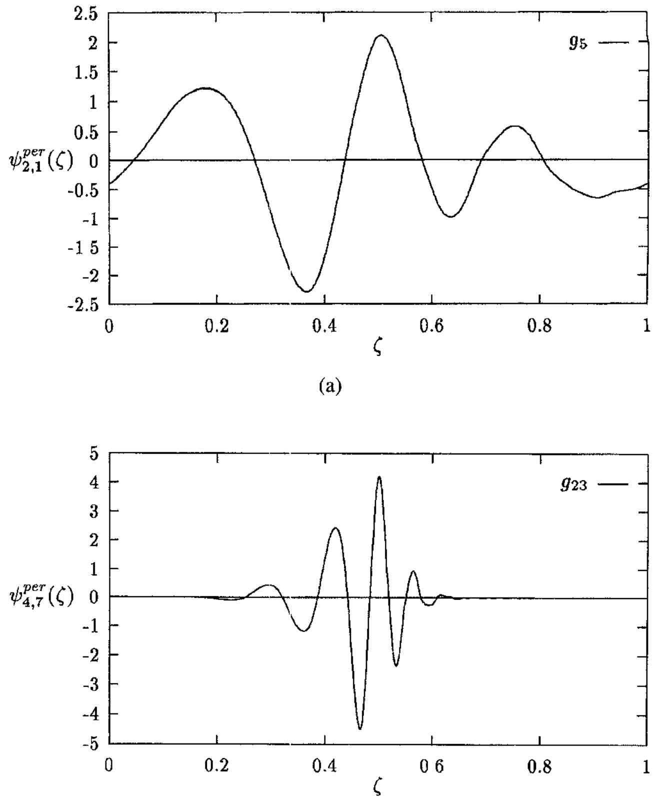

Fig. 1. The periodic wavelet constructed from the Daubechies orthogonal wavelet on the real line with $N = 7$. (a) $g_5(\zeta)\,[\psi^{per}_{2,1}(\zeta)]$ and (b) $g_{23}(\zeta)\,[\psi^{per}_{4,7}(\zeta)]$.

On the Utilization of Periodic Wavelet Expansions in the Moment Methods

Gaofeng Wang

***Abstract*—In this short paper, a new wavelet approach that makes use of periodic wavelet expansions in the moment methods is presented. The unknown field or response is expanded in terms of the periodic wavelet functions. As a wavelet expansion method, the moment-method matrix is rendered sparsely populated after applying a threshold procedure. Moreover, this approach circumvents the difficulties in the application of the conventional wavelet expansions on the real line to finite interval problems. Numerical study shows that this approach gives better accuracy than the use of the conventional wavelet expansions on the whole real line.**

I. Introduction

Recently, the wavelet expansion methods have been introduced to the applications of numerical analysis in electromagnetics (e.g., see [1]–[3]). Although the theory of wavelets is a relatively new area in mathematics, it has found many applications in engineering areas due to the special properties of wavelets. As a basis, the wavelet can be employed to express the unknown function in a series of wavelet functions. The wavelet expansion can adaptively fit itself to the various length scales associated with the physical configuration under study by distributing the localized basis functions near the discontinuities and the more spatially diffused ones over the smooth regions. Moreover, the cancellation property of the wavelets can eliminate, to a great extent, the coupling between the distant parts of the physical configuration under consideration. Attributed to these properties, the moment-method matrix obtained by a wavelet expansion is rendered sparsely populated as shown in [1]–[3].

However, difficulties exist when the unknown function is defined in finite intervals, while most of the wavelets are developed on the whole real line. In [1], Steinberg and Leviatan applied the Battle–Lemarie wavelet expansion on the real line to the moment method for solving an electromagnetic coupling problem [1]. Due to the infinite support of the Battle–Lemarie wavelet, the wavelet functions must be truncated to fit in the finite definition interval of the unknown function. The truncated wavelet basis lacks completeness over the finite interval under consideration. As a consequence, artificial oscillations appear in the results (e.g., see the magnitude of the equivalent magnetic current obtained from the truncated Battle–Lemarie wavelet in Fig. 4 of [1]).

A full wave analysis of microstrip floating line structures by wavelet expansion method was presented in [2], [3], where a Sommerfeld-type integral with an intractable kernel (the dyadic Green's functions for the grounded dielectric slab) was treated by using Daubechies wavelet. Since the Daubechies wavelet has compact support, one can easily delete the wavelet or scaling functions that are beyond the regions of interest, and thus the truncation of the

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